Name: ______________________________
Date: ______________________________
Instructor: Karyn I. Brown

AP Calculus BC
Summer 2011 Assignment
AB Review Material

Summer Assignment? Noooooooooo!!!!! Let me take a minute and explain the purpose, my expectation, the due date and the method of assessment as it relates to this assignment. "How come?" is one question that pops in your head. Over the past couple of years in BC Calculus, I've had to start back at the very beginning of AB (Chapter 1 – Limits and Continuity), re-explain every concept, re-assign all sections of homework, etc., etc., etc. This takes up a tremendous amount of classroom time and basically puts us behind in the content and curriculum that must be covered in order to be successful on the BC Exam.

I will begin the school year by addressing any questions over the summer assignment, collecting the summer assignment for a grade and then administering an exam over its content. We will then move directly into methods of integration – some methods you will have covered in AB and other methods will be new to you. Integration plays an extremely important role in the BC curriculum. After integration and its various applications, we will begin Chapter 8 – L'Hopital's Rule, Improper Integrals, and Growth Rates of Functions. This marks the beginning of the BC Calculus curriculum. It is during this chapter that we will "re-visit" the limits, continuity, and derivatives you have studied in AB Calculus.

The summer assignment has two purposes. The first is for me to find out what you know and don't know; that is why it is so important that you do your own work. The second is to help keep your knowledge of limits, derivatives, and their applications "up to snuff." It is only through the polishing and placing of these skills in your long-term memory that we can proceed through the course as described above. Also note that the BC Calculus exam covers 60% AB material and 40% BC material.

My expectations for this summer assignment are as follows:

1. You will complete the 50 multiple-choice questions.
2. You must show all work that is needed to complete each problem along with a written justification explaining how you arrived at your answer(s) on your own paper and circle the answer on the worksheet. Indicate in writing what you did in order to "solve the problem." What steps/methods did you use? What theorems, definitions and properties did you used to solve the problem? After completing the entire problem, write a statement indicating how comfortable you were in answering the question. Indicate what parts of the problem gave you difficulty and why.
3. You must have this assignment completed and ready to submit to me on day one of class. This assignment will prepare you for the exam I will be giving you at the beginning of the school year.
4. You must read all directions carefully and follow them very closely.
5. If you have any questions regarding the assignment, you will either e-mail me or call me over the summer. My e-mail address is karyn.brown@lrsd.org, my home phone number is 228-0903 and my cell is 352-9564. Please do not hesitate to call me with any questions regarding this assignment.

1.

x	-8	-6	-4	-2	0	2	4
$f(x)$	0	5	0	-2	-4	-6	-4
$f'(x)$	4	0	-4	-2	-1	0	1
$f''(x)$	-2	-6	-2	0	1	4	3

For some key values of x, the values of $f(x)$, $f'(x)$ and $f''(x)$ are given in the table above. The equation of the tangent to the curve $y = f(x)$ at the point of inflection shown in the table is:

(A) $y = 4x$ (B) $y = 4x + 8$
(C) $y = -6x + 24$ (D) $y = -2x - 6$ (E) $y = -x + 3$

2. The function $f(x)$ is continuous on the closed interval $[-3,5]$ and differentiable on the open interval $(-3,5)$. If $f'(x) > 0$ over the interval and if $f(-3) = -4$ and $f(5) = 12$, then $f(-1)$ cannot equal

(A) -6 (B) -1 (C) 4 (D) 5 (E) 10

3. Let $f(x) = \begin{cases} bx^2 + 6x, \text{ if } x \le 2 \\ ax^3, \text{ if } x > 2 \end{cases}$. Find the values of a and b such that $f(x)$ is differentiable at $x = 2$.

4. Determine the constants a, b, c, and d so that the graph of $y = ax^3 + bx^2 + cx + d$ has a relative maximum at (2, 4) and a point of inflection at the origin.

5. Let $f(x) = \begin{cases} \dfrac{x^3 + 8}{x + 2}, \text{ if } x \ne -2 \\ 4, \text{ if } x = -2 \end{cases}$. Which of the following four statements are true?

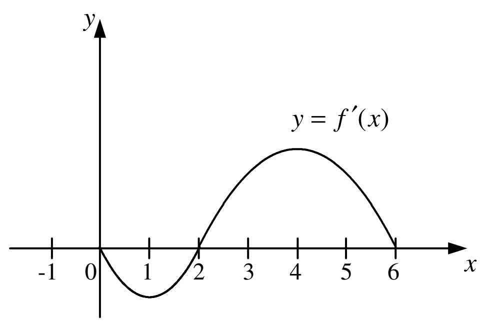
I. $f(x)$ is defined at $x = -2$
II. $f(x)$ is continuous at $x = -2$
III. $\lim_{x \to -2} f(x)$ exists
IV. $f(x)$ is differentiable at $x = -2$

(A) I only (B) I and II (C) I and III
(D) II and III (E) I, II, III and IV

6. For a right circular cylinder with radius r and height h, volume $V = \pi r^2 h$ and surface area $S = 2\pi r^2 + 2\pi rh$. If the radius is a function of time and the height of the cylinder is equal to the diameter, then $\frac{dV}{dt} =$

(A) $r\frac{dS}{dt}$ (B) $2r\frac{dS}{dt}$ (C) $\frac{r}{2}\frac{dS}{dt}$ (D) $\frac{1}{r}\frac{dS}{dt}$ (E) $\frac{1}{r^2}\frac{dS}{dt}$

7.

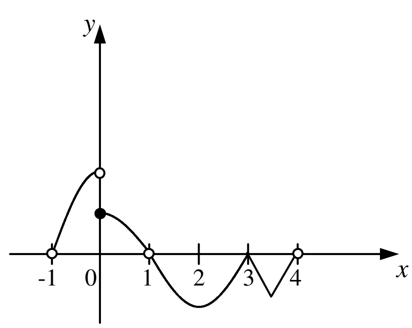


The graph of $f'(x)$ is shown above. For which of the following values of x is $f(x)$ concave down?

(A) $x = \frac{1}{2}$ (B) $x = \frac{3}{2}$ (C) $x = 2$ (D) $x = \frac{5}{2}$ (E) $x = 3$

8. The acceleration of a particle is given by $a(t) = 36t^2 - 12$ and $s(t)$ is the position function. If $s(-1) = -2$ and $s(2) = 37$, find the velocity of the particle at $t = 1$.

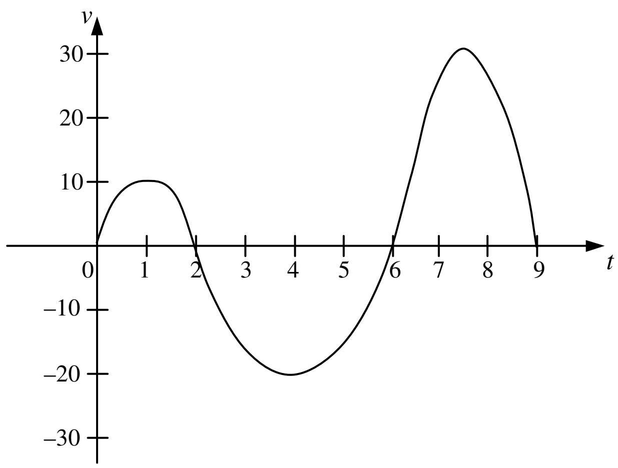
9.



The function, shown above, is differentiable at

(A) $x = 1$ (B) $x = 2$ (C) $x = 3$ (D) $x = 2$ and $x = 3$
(E) $x = 0$, $x = 3$, and $x = 4$

10.



The graph of the velocity of a particle traveling along the x-axis for $0 \le t \le 9$ is shown above. At $t = 0$, $x = 0$. Approximately how many units to the right of the origin is the position of the particle at $t = 9$?

(A) 2 (B) 8 (C) 15 (D) 27 (E) 31

11. The average value of the function $f(x) = ax^2 - 2ax + a$ over the interval [1, 4] equals 13. Find the value of a.

12. If the function $f(x)$ is such that $f(1) = 4$, $f(2) = 4$, and $f''(x)$ exists and is positive on the closed interval [0, 4], then we must have

(A) $f'(1.5) = 0$ (B) $f'(1.5) > 0$
(C) $f'(3) > 0$ (D) $f'(3) < 0$
(E) none of the above

13. If $f'(x) = 6x^2$ and $f(2) = 1$, then $\int_0^2 f(x)\,dx =$

(A) -22 (B) -16 (C) 2 (D) 8 (E) 18

14. The expression $e^1 + e^2 + e^3$ is a

(A) left-hand Riemann sum with 3 subintervals for $\int_0^4 e^x dx$

(B) left-hand Riemann sum with 3 subintervals for $\int_0^3 e^x dx$

(C) right-hand Riemann sum with 3 subintervals for $\int_0^3 e^x dx$

(D) right-hand Riemann sum with 3 subintervals for $\int_1^4 e^x dx$

(E) midpoint Riemann sum with 4 subintervals for $\int_0^4 e^x dx$

15. The line $y = 4x + 13$ is tangent to the curve $y = -2x^2 + kx + 5$, where $k < 0$. Find *k*.

16. Let $f(x) = \begin{cases} x + 2a, & \text{if } x < 1 \\ ax^2 + 7x - 4, & \text{if } x \geq 1 \end{cases}$. If *a* is such that $f(x)$ is continuous at $x = 1$, is $f(x)$ also differentiable at $x = 1$? Justify your answer.

17. Let *f* and *g* be differentiable functions, where $f(2) = 6$, $g(2) = 4$, $f'(2) = -5$, $g'(2) = -2$, $f'(4) = -3$, $g'(4) = 3$. If $h(x) = f\left(g(x)\right)$, then $h'(2) =$

(A) -20 (B) -12 (C) -6 (D) -3 (E) 6

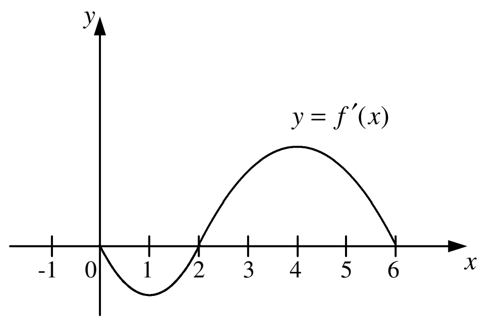
18. $\lim_{h \to 0} \frac{e^{4+h} - e^4}{h} =$

(A) e^3 (B) e^4 (C) $4e^3$ (D) $4e^4$ (E) $5e^5$

19. If $f(x)$ is an odd function, $\int_0^3 f(x)\,dx = 8$, and $\int_2^3 f(x)\,dx = 2$, then $\int_{-2}^0 f(x)\,dx =$

(A) -10 (B) -6 (C) 0 (D) 6 (E) 10

20.



The graph of $f'(x)$ is shown above. On the interval $0 \le x \le 6$, for what value of x does $f(x)$ achieve its absolute maximum value?

(A) $x = 0$ (B) $x = 2$ (C) $x = 4$ (D) $x = 6$
(E) cannot be determined

21. Find the value of c which satisfies Rolle's Theorem for the function $f(x) = \sin\left(x^2\right)$ on $\left[0, \sqrt{\pi}\right]$.

22. If $y = 3\sin x + 4\cos x$, then $y'' - y =$

(A) $-6\sin x - 8\cos x$ (B) $-6\sin x + 8\cos x$
(C) $6\sin x - 8\cos x$ (D) $6\sin x + 8\cos x$
(E) 0

23. If $f(x) = x\ln x$, $x > 0$, then $f'(x) < 0$ when

(A) $0 < x < \frac{1}{e}$ (B) $\frac{1}{e} < x < 1$ (C) $1 < x < e$ (D) $x > 0$ (E) $x > e$

24. ▦ If $\int_1^2 (4 + \ln x)\, dx$ is approximated by a midpoint Riemann sum with four subintervals of equal length, then the value is

(A) 4.297 (B) 4.388 (C) 4.470 (D) 4.514 (E) 4.669

25. $\lim_{x\to\infty} \frac{\sqrt{x} + \sqrt[3]{8x} + 4}{\sqrt{4x} + \sqrt[3]{x} + 4} =$

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) $\frac{5}{2}$

26. If $f(x)=\left(4x^3-4\right)\left(\sqrt{x}-2\right)$, then $f'(1)=$

(A) -12 (B) $-11\frac{1}{2}$ (C) 0 (D) 1 (E) $11\frac{1}{2}$

27. An object is thrown vertically into the air. Its height in feet at any time t is given by $h(t)=-16t^2+48t+5$. How high does the object rise?

(A) 37 ft (B) 41 ft (C) 48 ft (D) 53 ft (E) 113 ft

28. If $f(x)=|x|$, then $f'(-2)=$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

29. If $f(x)=\frac{1}{(1-x)^2}$, then the n-th derivative of $f(x)$ is

(A) $\frac{n!}{(1-x)^n}$ (B) $\frac{(n+1)!}{(1-x)^{n+1}}$ (C) $\frac{(n+1)!}{(1-x)^{n+2}}$
(D) $\frac{(-1)^n(n+1)!}{(1-x)^{n+2}}$ (E) $\frac{(-1)^{n+1}(n+1)!}{(1-x)^{n+1}}$

30. Three tangent lines can be drawn to the curve $y=x^3+4x^2$ from the point $(1,-4)$. The sum of the slopes of these three tangent lines is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $15\frac{1}{2}$ (D) $20\frac{3}{4}$ (E) $28\frac{1}{4}$

31. $\lim\limits_{h\to 0}\frac{8\sqrt{16+h}-32}{h}=$

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

32. If $x^4-3x^2y^2+4y^2=5$, then the value of $\frac{dy}{dx}$ at (1, 2) is

(A) -1 (B) 1 (C) 3 (D) 5 (E) undefined

33. A particle moves along the x-axis with acceleration $a(t)=6t-4$ units^2/sec. Given that $x=10$ when $t=1$ and $x=30$ when $t=3$, find the velocity of the particle at $t=2$.

34. The curve $3y^2 - 3xy + 2x^3 = 7$ has vertical tangents when

(A) $x = y$ (B) $2x = y$ (C) $x = 2y$ (D) $3x = y$ (E) $x = 3y$

35. If $\int_3^5 f(x)\,dx = k$, then $\int_3^5 f(x)\,dx - \int_3^1 f(x+2)\,dx =$

(A) $-2k$ (B) $-k$ (C) 0 (D) k (E) $2k$

36. If $y = e^{-x}\sin x$, then $\dfrac{d^2y}{dx^2} =$

(A) $-2e^{-x}\sin x$ (B) $-2e^{-x}\cos x$
(C) $-2e^{-x}(\sin x + \cos x)$ (D) $e^{-x}\sin x\cos x$
(E) $-2e^{-x}\sin x\cos x$

37. If $y = \dfrac{\ln x}{\sin x}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{\cos x - \sin x \ln x}{\sin^2 x}$ (B) $\dfrac{\sin x - x\cos x \ln x}{x\sin^2 x}$
(C) $\dfrac{x\sin x - \cos x \ln x}{x\sin^2 x}$ (D) $\dfrac{\cos x - x\sin x \ln x}{x\cos^2 x}$
(E) $\dfrac{\sin x}{x} - \dfrac{\ln x}{\cos x}$

38. $\int_a^3 |x+1|\,dx$, where $a < -1$, is equal to

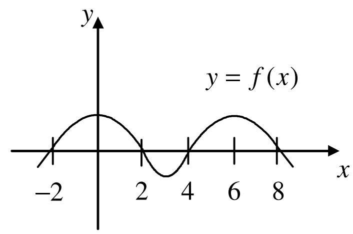
(A) $a - \dfrac{a^2}{2} + \dfrac{15}{2}$ (B) $a - \dfrac{a^2}{2} + \dfrac{17}{2}$
(C) $\dfrac{a^2}{2} - a + \dfrac{15}{2}$ (D) $\dfrac{a^2}{2} + a + \dfrac{17}{2}$
(E) none of the above

39. If $f(x) = x^2 \ln x$, then $f'(x) < 0$ for all

(A) $x < 0$ (B) $x > 0$
(C) $0 < x < \dfrac{1}{\sqrt{e}}$ (D) $0 < x < e$
(E) $0 < x < \sqrt{e}$

40. A rectangle is inscribed into the region bounded by the graph of $f(x)=\left(x^2-1\right)^2$ and the x-axis, in such a way that one side of the rectangle lies on the x-axis and two vertices lie on the graph of $f(x)$. What is the maximum possible area of such a rectangle?

41.



The graph of $y=f(x)$ is shown above. Over what interval(s) is the derivative of $f(x)$ increasing?

(A) $-2\le x\le 0$ and $4\le x\le 6$
(B) $2\le x\le 4$
(C) $0\le x\le 3$ and $6\le x\le 8$
(D) $-2\le x\le 2$ and $4\le x\le 8$
(E) $-2\le x\le 8$

42. A particle is moving in a straight line with an acceleration $a=\dfrac{dv}{dt}=-v^2$. At $t=0$, $v=1$. The distance covered from $t=1$ to $t=5$ is

(A) e^2 (B) $\ln 2$ (C) $\ln 3$ (D) $\ln 4$ (E) $\ln 12$

43. If $f(x)=\int_{-10}^{x^2}\sqrt{1+t^2}\,dt$, then $f'(2)=$

(A) -10 (B) $\sqrt{5}$ (C) $2\sqrt{5}$ (D) $\sqrt{17}$ (E) $4\sqrt{17}$

44. The average value of the function $y=4\cos 2x$ from $x=0$ to $x=\dfrac{\pi}{4}$ is

(A) $\dfrac{2}{\pi}$ (B) $\dfrac{4}{\pi}$ (C) $\dfrac{8}{\pi}$ (D) 2 (E) 4

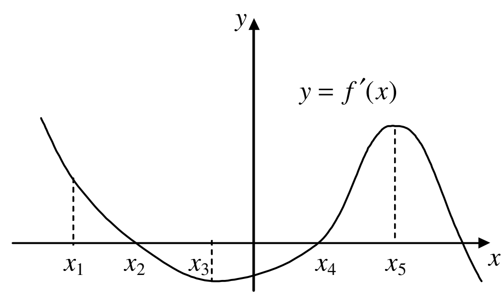
45. $\int_1^4 \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\,dx=$

(A) $2e(e-1)$ (B) $2e(e+1)$ (C) $5(e-1)$
(D) $3e^2-e^3+7$ (E) e^3-2e^2+4

46. If $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=g(x)$, then

(A) $f(x)=g(x)$ (B) $f'(x)=g(x)$
(C) $f(x)=g'(x)$ (D) $f'(x)=g'(x)$
(E) $\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}=f(x)$

47.



The figure above shows a graph of $f'(x)$. At the point $(x_1, f(x_1))$, the curve $y=f(x)$ is

(A) rising and concave up
(B) rising and concave down
(C) falling and concave up
(D) falling and concave down
(E) neither rising nor falling

48. The curve $y=x^5+10x^4-5$ has points of inflection at $x=$

(A) 0 and -8 (B) 0 and -6 (C) -8 only
(D) -6 only (E) 0 only

49. The area enclosed by the graph of $y=x^2$ and the graph of its inverse function is

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

50. The slope of the line normal to the curve $y=xe^x$ at $x=-1$ is

(A) 0 (B) $\frac{2}{e}$ (C) $-\frac{e}{2}$ (D) e (E) undefined