To: AP Calculus BC Students –
2009 – 2010 school year
From: Karyn I. Brown, Instructor
Date: June 2, 2009
Re: Summer 2009 Assignment

Summer Assignment? Noooooooooo!!!!! Let me take a minute and explain the purpose, my expectation, the due date and the method of assessment as it relates to this assignment. "How come?" is one question that pops in your head. Over the past couple of years in BC Calculus, I've had to start back at the very beginning of AB (Chapter 1 – Limits and Continuity), re-explain every concept, re-assign all sections of homework, etc., etc., etc. This takes up a tremendous amount of classroom time and basically puts us behind in the content and curriculum that must be covered in order to be successful on the BC Exam.

I will begin the school year by addressing any questions over the summer assignment, collecting the summer assignment and then administering a diagnostic exam over its content. We will then move directly into methods of integration – some methods you will have covered in AB and other methods that will be new to you. Integration plays an extremely important role in the BC curriculum. After integration and its various applications, we will begin Chapter 8 – L'Hopital's Rule, Improper Integrals, and Growth Rates of Functions. This marks the beginning of the BC Calculus curriculum.

The summer assignment has two purposes. The first is for me to find out what you know and don't know; that is why it is so important that you do your own work. The second is to help keep your knowledge of limits, derivatives, and their applications "up to snuff." It is only through the polishing and placing of these skills in your long-term memory that we can proceed through the course as described above. Also note that the BC Calculus exam covers 60% AB material and 40% BC material. The AP Calculus BC exam is scheduled for May 6th, 2010.

My expectations for this summer assignment are as follows:

1. You will complete the six open-response questions in the assignment following all directions.
2. You must show all work that is needed to complete each problem along with a written justification explaining how you arrived at your answer(s). Indicate in writing what you did in order to "solve the problem." What steps/methods did you use? What theorems, definitions and properties did you used to solve the problem? After completing the entire problem, write a statement indicating how comfortable you were in answering the question. Indicate what parts of the problem gave you difficulty and why.
3. You must have this assignment completed and ready to submit to me on day one of class. This assignment will prepare you for the diagnostic exam I will be giving you at the beginning of the year.
4. You must read all directions carefully and follow them very closely.
5. If you have any questions regarding the assignment, you will either e-mail me or call me over the summer. My e-mail address is karyn.brown@lrsd.org, my home phone number is 228-0903 and my cell is 551-3480. Please do not hesitate to call me with any questions regarding this assignment.

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Open-Response Questions

Directions: In working problems 1, 2 and 3 – a graphing calculator is required. You are to show all work that is needed to complete each problem. Remember to justify your answer(s) by indicating any methods, theorems, definitions and properties used. Once you have completed the problem and the justification, write a statement indicating how comfortable you were in answering the question and what parts of the problem gave you difficulty and why. Make sure you show all work and justification in the space provided. When reflecting on the comfort level you had with respect to the question, please write on your own notebook paper and attach it to the respective problem.

1. At a certain height, a tree trunk has a circular cross section. The radius R(t) of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3+\sin(t^2)\right) \text{ centimeters per year}$$

For $0 \le t \le 3$, where time t is measured in year. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

a. Write an expression, involving an integral, for the radius *R(t)* for $0 \le t \le 3$.

b. Use your expression from part a to find *R(3)*.

c. Find the rate at which the cross-sectional area *A(t)* is increasing at time t = 3 years. Indicate the units of measure.

d. Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

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2. **A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t)=\sqrt{t}+\cos t-3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began. The storm lasted a total of 5 hours. The derivative of $f(t)=\sqrt{t}+\cos t-3$ is $f'(t)=\frac{1}{2\sqrt{t}}-\sin t.$**

a. What was the distance between the road and the edge of the water at the end of the storm?

b. Using correct units, interpret the value $f'(4)=1.007$ in terms of the distance between the road and the edge of the water.

c. At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly?

d. After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

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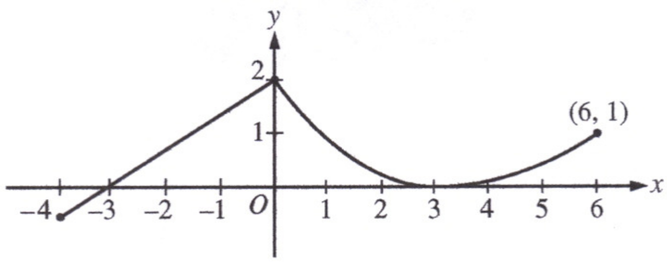

Graph of f

3. **A continuous function *f* defined on the closed interval $-4 \le x \le 6$. The graph of *f* consists of a line segment and a curve that is tangent to the *x*-axis at *x* = 3, as shown in the figure above. On the interval $0 < x < 6$, the function *f* is twice differentiable, with** $f''(x) > 0$.

a. If *f* differentiable at $x = 0$? **NOTE:** Use the definition of the derivative with one-sided limits to justify your answer.

b. For how many values of *a*, $-4 \le a < 6$, is the average rate of change of *f* on the interval [a, 6] equal to 0? Justify your answer(s).

c. Is there a value of *a*, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval [a, 6], guarantees a value *c*, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

d. The function of *g* is defined by $g(x) = \int_0^x f(t)dt$ for $-4 \le x \le 6$. On what intervals contained in [-4, 6] is the graph of *g* concave up? Justify your answer(s).

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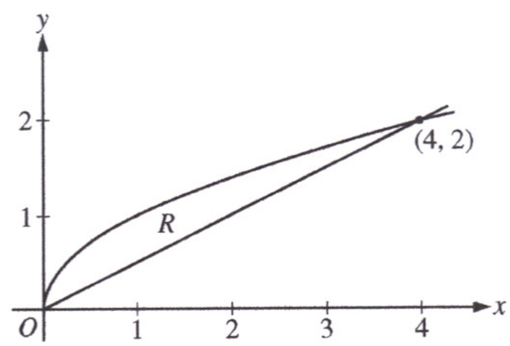
Directions: In working problems 4, 5 and 6 – no calculator is allowed. You are to show all work that is needed to complete each problem. Remember to justify your answer(s) by indicating any methods, theorems, definitions and properties used. Once you have completed the problem and the justification, write a statement indicating how comfortable you were in answering the question and what parts of the problem gave you difficulty and why. Make sure you show all work and justification in the space provided. When reflecting on the comfort level you had with respect to the question, please write on your own notebook paper and attach it to the respective problem.



4. **Let *R* be the region bounded by the graphs of** $y = \sqrt{x}$ and $y = \frac{x}{2}$ **, as shown in the figure above.**

a. Find the area of the region *R*.

b. The region *R* is the base of a solid. For this solid, the cross sections perpendicular to the *x*-axis are squares. Find the volume of this solid.

c. Write, but do not evaluate, an integral expression for the volume of the solid generated when *R* is rotated about the horizontal line $y = 2$.

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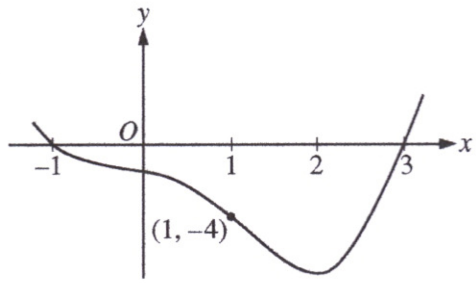

Graph of f'

5. **Let *f* be a twice-differentiable function defined on the interval** $-1.2 < x < 3.2$ **with** $f(1) = 2$. **The graph of** f'**, the derivative of *f*, is shown above. The graph of** f' **crosses the *x*-axis at *x* = -1 and *x* = 3 and has a horizontal tangent at *x* = 2. Let *g* be the function given by** $g(x) = e^{f(x)}$.

a. Write an equation for the line tangent to the graph of g at $x = 1$.

b. **For** $-1.2 < x < 3.2$, **find all values of *x* at which *g* has a local maximum. Justify your answer.**

c. **The second derivative of *g* is** $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$.

Is $g''(-1)$ **positive, negative, or zero? Justify your answer.**

d. **Find the average rate of change of** g'**, the derivative of** g**, over** $[1,3]$.

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t (seconds)	0	8	20	25	32	40
v(t) (meters per second)	3	5	-10	-8	-4	7

6. The velocity of a particle moving along the x-axis is modeled by a differentiable functions v, where the position x is measured in meters and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

a. Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure and justify your answer.

b. Using correct units, explain the meaning of $\int_{20}^{40} v(t)dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)dt$. Show all work and justify your answer.

c. For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

d. Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.